

PHYS 480/581 General Relativity

Worksheet #5
Monday September 21 2026

Question 1.

Consider a sphere of radius R with coordinates θ, ϕ , where θ is the angle from the positive z axis (as is conventional in physics). The metric takes the form

$$g_{\mu\nu} = \begin{bmatrix} R^2 & 0 \\ 0 & R^2 \sin^2 \theta \end{bmatrix}. \quad (1)$$

Here we want to solve the geodesic equation on this surface

$$\frac{d^2 x^\alpha}{ds^2} + \Gamma_{\mu\nu}^\alpha \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = 0, \quad (2)$$

where ds stands for an infinitesimal length segment on the surface of the sphere and

$$\Gamma_{\mu\nu}^\alpha \equiv \frac{1}{2} g^{\alpha\sigma} (\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}). \quad (3)$$

- (a) Compute the Christoffel connection $\Gamma_{\theta\theta}^\theta$, $\Gamma_{\phi\theta}^\theta$, $\Gamma_{\phi\phi}^\theta$, $\Gamma_{\phi\phi}^\phi$, $\Gamma_{\theta\phi}^\phi$, and $\Gamma_{\theta\theta}^\phi$.
- (b) Derive the $\alpha = \theta$ and $\alpha = \phi$ components of the geodesic equation.
- (c) Show that the equator (the curve with $\theta = \pi/2$ for any ϕ) is a solution to the above geodesic equation.