

PHYS 480/581 General Relativity

Worksheet #6
Monday September 28 2026

Question 1.

In the first week of class, we discussed the Equivalence Principle. One version of this principle was that the laws of physics reduce to those of Special Relativity, *in a small enough region of spacetime*. We now have the technology to make this statement rigorous.

Since Special Relativity is characterized by the Minkowski metric $\eta_{\mu\nu}$, the Equivalence Principle implies that we can always find a coordinate transformation $x \rightarrow x'(x)$ such that the metric and its first derivatives *at a given point* p in the primed frame are

$$g'_{\mu\nu}(p) = \eta_{\mu\nu} \quad \text{and} \quad \partial_\alpha g'_{\mu\nu}(p) = 0. \quad (1)$$

If the above is true, then the x' coordinates are called *Local Inertial Coordinates*, and they characterized a local inertial frame. Note that in general it *not* possible to set the second derivative of the metric to zero at p .

(a) To set the metric to Minkowski form, we only have access to the Jacobian of the $x \rightarrow x'(x)$ transformation. By counting the number of independent elements in $g_{\mu\nu}$ and in the Jacobian $\partial x^\alpha / \partial x'^\mu$, argue that there is enough freedom to set $g'_{\mu\nu} = \eta_{\mu\nu}$. To set $\partial_\alpha g'_{\mu\nu}(p) = 0$, we can use the freedom of the second derivative of the coordinate transformation $\partial^2 x^\alpha / \partial x'^\mu \partial x'^\nu$. Argue that there is enough freedom in this tensor to set all the first derivatives of the metric to zero.

(b) Show that the Riemann tensor take the simple form

$$R_{\alpha\beta\mu\nu} = \frac{1}{2} (\partial_\beta \partial_\mu g_{\alpha\nu} + \partial_\alpha \partial_\nu g_{\beta\mu} - \partial_\beta \partial_\nu g_{\alpha\mu} - \partial_\alpha \partial_\mu g_{\beta\nu}) \quad (2)$$

in local inertial coordinates.