PHYS 480/581
General Relativity

Worksheet #7

Wednesday October 21 2026

Question 1.

Consider the diagonal metric

$$ds^2 = -A\,(dx^0)^2 + B\,(dx^1)^2 + C\,(dx^2)^2 + D\,(dx^3)^2, \tag{1}$$

where x^0, x^1, x^2, and x^3 are completely arbitrary coordinates. A, B, C, and D are arbitrary functions of these coordinates. In the following, we will use the notation

$$A_0 \equiv \frac{\partial A}{\partial x^0},\ A_1 \equiv \frac{\partial A}{\partial x^1},\ B_{12} \equiv \frac{\partial^2 B}{\partial x^1 x^2},\ \text{etc.} \tag{2}$$

In the following three pages, you will find a worksheet to compute the entries of the Ricci tensor $R_{\mu\nu}$ for the above diagonal metric.

Here we would like to apply this worksheet to the computation of the Ricci tensor for the Schwarzschild metric

$$ds^2 = -A(t,r)\,dt^2 + B(t,r)\,dr^2 + r^2(d\theta^2 + \sin^2\theta\,d\phi^2), \tag{3}$$

where we have taken our coordinates to be $x^0 = t$, $x^1 = r$, $x^2 = \theta$, and $x^3 = \phi$, and $A = A(t,r)$, $B = B(t,r)$, $C = r^2$, and $D = r^2\sin^2\theta$. Use the worksheet below to compute all entries of the Ricci tensor. For instance, using the R_{12} entry on page 4 below,

$$\begin{aligned} R_{12} = R_{r\theta} &= -\frac{1}{2A}A_{12} - \frac{1}{2D}D_{12} + \frac{1}{4A^2}A_1A_2 + \frac{1}{4D^2}D_1D_2 + \frac{1}{4BA}B_2A_1 + \frac{1}{4BD}B_2D_1 \\ &\quad + \frac{1}{4CA}C_1A_2 + \frac{1}{4CD}C_1D_2 \\ &= 0 - \frac{4r\sin\theta\cos\theta}{2r^2\sin^2\theta} + 0 + \frac{4r\sin^2\theta\, r^2\sin\theta\cos\theta}{4r^4\sin^4\theta} + 0 + 0 + 0 + \frac{4r^3\sin\theta\cos\theta}{4r^4\sin^2\theta} \\ &= -\frac{2\cos\theta}{r\sin\theta} + \frac{\cos\theta}{r\sin\theta} + \frac{\cos\theta}{r\sin\theta} \\ &= 0. \end{aligned} \tag{4}$$

Using the worksheet below, repeat this calculation for the other components of the Ricci tensor. Start by R_{tt}, R_{rr}, $R_{\theta\theta}$, and R_{tr}.

$$
\begin{aligned}
R_{00} = {} & 0 + \tfrac{1}{2B}A_{11} + \tfrac{1}{2C}A_{22} + \tfrac{1}{2D}A_{33} \\
& + 0 - \tfrac{1}{2B}B_{00} - \tfrac{1}{2C}C_{00} - \tfrac{1}{2D}D_{00} \\
& + 0 + \tfrac{1}{4B^2}B_0^2 + \tfrac{1}{4C^2}C_0^2 + \tfrac{1}{4D^2}D_0^2 \\
& + 0 + \tfrac{1}{4AB}A_0B_0 + \tfrac{1}{4AC}A_0C_0 + \tfrac{1}{4AD}A_0D_0 \\
& - \tfrac{1}{4BA}A_1A_1 - \tfrac{1}{4B^2}A_1B_1 + \tfrac{1}{4BC}A_1C_1 + \tfrac{1}{4BD}A_1D_1 \\
& - \tfrac{1}{4CA}A_2A_2 + \tfrac{1}{4CB}A_2B_2 - \tfrac{1}{4C^2}A_2C_2 + \tfrac{1}{4CD}A_2D_2 \\
& - \tfrac{1}{4DA}A_3A_3 + \tfrac{1}{4DB}A_3B_3 + \tfrac{1}{4DC}A_3C_3 - \tfrac{1}{4D^2}A_3D_3
\end{aligned}
$$

$R_{00} =$

$$
\begin{aligned}
R_{11} = {} & \tfrac{1}{2A}B_{00} + 0 - \tfrac{1}{2C}B_{22} - \tfrac{1}{2D}B_{33} \\
& - \tfrac{1}{2A}A_{11} + 0 - \tfrac{1}{2C}C_{11} - \tfrac{1}{2D}D_{11} \\
& + \tfrac{1}{4A^2}A_1^2 + 0 + \tfrac{1}{4C^2}C_1^2 + \tfrac{1}{4D^2}D_1^2 \\
& - \tfrac{1}{4A^2}B_0A_0 - \tfrac{1}{4AB}B_0B_0 + \tfrac{1}{4AC}B_0C_0 + \tfrac{1}{4AD}B_0D_0 \\
& + \tfrac{1}{4BA}B_1A_1 + 0 + \tfrac{1}{4BC}B_1C_1 + \tfrac{1}{4BD}B_1D_1 \\
& - \tfrac{1}{4CA}B_2A_2 + \tfrac{1}{4CB}B_2B_2 + \tfrac{1}{4C^2}B_2C_2 - \tfrac{1}{4CD}B_2D_2 \\
& - \tfrac{1}{4DA}B_3A_3 + \tfrac{1}{4DB}B_3B_3 - \tfrac{1}{4DC}B_3C_3 + \tfrac{1}{4D^2}B_3D_3
\end{aligned}
$$

$R_{11} =$

$$
\begin{aligned}
R_{22} = {} & \tfrac{1}{2A} C_{00} && - \tfrac{1}{2B} C_{11} && + 0 && - \tfrac{1}{2D} C_{33} \\
& - \tfrac{1}{2A} A_{22} && - \tfrac{1}{2B} B_{22} && + 0 && - \tfrac{1}{2D} D_{22} \\
& + \tfrac{1}{4A^2} A_2^2 && + \tfrac{1}{4B^2} B_2^2 && + 0 && + \tfrac{1}{4D^2} D_2^2 \\
& - \tfrac{1}{4A^2} C_0 A_0 && + \tfrac{1}{4AB} C_0 B_0 && - \tfrac{1}{4AC} C_0 C_0 && + \tfrac{1}{4AD} C_0 D_0 \\
& - \tfrac{1}{4BA} C_1 A_1 && + \tfrac{1}{4B^2} C_1 B_1 && + \tfrac{1}{4BC} C_1 C_1 && - \tfrac{1}{4BD} C_1 D_1 \\
& + \tfrac{1}{4CA} C_2 A_2 && + \tfrac{1}{4CB} C_2 B_2 && + 0 && + \tfrac{1}{4CD} C_2 D_2 \\
& - \tfrac{1}{4DA} C_3 A_3 && - \tfrac{1}{4DB} C_3 B_3 && + \tfrac{1}{4DC} C_3 C_3 && + \tfrac{1}{4D^2} C_3 D_3
\end{aligned}
$$

$R_{22} =$

$$
\begin{aligned}
R_{33} = {} & \tfrac{1}{2A} D_{00} && - \tfrac{1}{2B} D_{11} && - \tfrac{1}{2C} D_{22} && + 0 \\
& - \tfrac{1}{2A} A_{33} && - \tfrac{1}{2B} B_{33} && - \tfrac{1}{2C} C_{33} && + 0 \\
& + \tfrac{1}{4A^2} A_3^2 && + \tfrac{1}{4B^2} B_3^2 && + \tfrac{1}{4C^2} C_3^2 && + 0 \\
& - \tfrac{1}{4A^2} D_0 A_0 && + \tfrac{1}{4AB} D_0 B_0 && + \tfrac{1}{4AC} D_0 C_0 && - \tfrac{1}{4AD} D_0 D_0 \\
& - \tfrac{1}{4BA} D_1 A_1 && + \tfrac{1}{4B^2} D_1 B_1 && - \tfrac{1}{4BC} D_1 C_1 && + \tfrac{1}{4BD} D_1 D_1 \\
& - \tfrac{1}{4CA} D_2 A_2 && - \tfrac{1}{4CB} D_2 B_2 && + \tfrac{1}{4C^2} D_2 C_2 && + \tfrac{1}{4CD} D_2 D_2 \\
& + \tfrac{1}{4DA} D_3 A_3 && + \tfrac{1}{4DB} D_3 B_3 && + \tfrac{1}{4DC} D_3 C_3 && + 0
\end{aligned}
$$

$R_{33} =$

$$R_{01} = -\frac{1}{2C}C_{01} - \frac{1}{2D}D_{01} + \frac{1}{4C^2}C_0C_1 + \frac{1}{4D^2}D_0D_1$$
$$+\frac{1}{4AC}A_1C_0 + \frac{1}{4AD}A_1D_0 + \frac{1}{4BC}B_0C_1 + \frac{1}{4BD}B_0D_1$$

$R_{01} =$

$$R_{02} = -\frac{1}{2B}B_{02} - \frac{1}{2D}D_{02} + \frac{1}{4B^2}B_0B_2 + \frac{1}{4D^2}D_0D_2$$
$$+\frac{1}{4AB}A_2B_0 + \frac{1}{4AD}A_2D_0 + \frac{1}{4CB}C_0B_2 + \frac{1}{4CD}C_0D_2$$

$R_{02} =$

$$R_{03} = -\frac{1}{2B}B_{03} - \frac{1}{2C}C_{03} + \frac{1}{4B^2}B_0B_3 + \frac{1}{4C^2}C_0C_3$$
$$+\frac{1}{4AB}A_3B_0 + \frac{1}{4AC}A_3C_0 + \frac{1}{4DB}D_0B_3 + \frac{1}{4DC}D_0C_3$$

$R_{03} =$

$$R_{12} = -\frac{1}{2A}A_{12} - \frac{1}{2D}D_{12} + \frac{1}{4A^2}A_1A_2 + \frac{1}{4D^2}D_1D_2$$
$$+\frac{1}{4BA}B_2A_1 + \frac{1}{4BD}B_2D_1 + \frac{1}{4CA}C_1A_2 + \frac{1}{4CD}C_1D_2$$

$R_{12} =$

$$R_{13} = -\frac{1}{2A}A_{13} - \frac{1}{2C}C_{13} + \frac{1}{4A^2}A_1A_3 + \frac{1}{4C^2}C_1C_3$$
$$+\frac{1}{4BA}B_3A_1 + \frac{1}{4BC}B_3C_1 + \frac{1}{4DA}D_1A_3 + \frac{1}{4DC}D_1C_3$$

$R_{13} =$

$$R_{23} = -\frac{1}{2A}A_{23} - \frac{1}{2B}B_{23} + \frac{1}{4A^2}A_2A_3 + \frac{1}{4B^2}B_2B_3$$
$$+\frac{1}{4CA}C_3A_2 + \frac{1}{4CB}C_3B_2 + \frac{1}{4DA}D_2A_3 + \frac{1}{4DB}D_2B_3$$

$R_{23} =$