

# PHYS 480/581 General Relativity

## Homework Assignment 7

Due date: Monday 10/05/2026 8pm, submitted electronically on UNM Canvas

### Question 1 (5 points).

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We saw in class that the Riemann tensor is defined as the commutator of covariant derivatives

$$[\nabla_\mu, \nabla_\nu]V^\alpha = R^\alpha_{\beta\mu\nu}V^\beta, \quad (1)$$

where  $\mathbf{V}$  is an arbitrary 4-vector. Note that in flat spacetime, the covariant derivatives reduce to standard partial derivatives (which commute), implying that  $R^\alpha_{\beta\mu\nu} = 0$  in that case, as it should.

(a) Since  $\nabla_\mu V^\alpha$  is a standard (1,1) tensor, show that

$$\begin{aligned} \nabla_\mu(\nabla_\nu V^\alpha) &= \partial_\mu \partial_\nu V^\alpha + (\partial_\mu \Gamma^\alpha_{\beta\nu})V^\beta + \Gamma^\alpha_{\beta\nu} \partial_\mu V^\beta - \Gamma^\sigma_{\nu\mu} \partial_\sigma V^\alpha \\ &\quad - \Gamma^\sigma_{\nu\mu} \Gamma^\alpha_{\gamma\sigma} V^\gamma + \Gamma^\alpha_{\delta\mu} \partial_\nu V^\delta + \Gamma^\alpha_{\delta\mu} \Gamma^\delta_{\lambda\nu} V^\lambda. \end{aligned} \quad (2)$$

(b) Use the result from part (a) to derive the standard relationship between the Riemann curvature tensor and the Christoffel connection

$$R^\alpha_{\beta\mu\nu} = \partial_\mu \Gamma^\alpha_{\beta\nu} - \partial_\nu \Gamma^\alpha_{\beta\mu} + \Gamma^\alpha_{\mu\gamma} \Gamma^\gamma_{\beta\nu} - \Gamma^\alpha_{\nu\gamma} \Gamma^\gamma_{\beta\mu}. \quad (3)$$

### Question 2 (4 points).

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Consider the following metric for a two-dimensional space

$$ds^2 = dp^2 + e^{2p/p_0} dq^2, \quad (4)$$

where  $p_0$  is a constant with units of length.

(a) Argue that the only nonzero Christoffel symbols for this metric are  $\Gamma^p_{qq}$  and  $\Gamma^q_{pq} = \Gamma^q_{qp}$ , and evaluate these symbols.

(b) Is this space curved? Prove your assertion.

### Question 3 (5 points).

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Consider the metric

$$ds^2 = -dt^2 + a(t)^2 [dx^2 + dy^2 + dz^2], \quad (5)$$

where  $a(t)$  is an increasing function of time (and  $x, y, z$  are Euclidean spatial coordinates), which describes a homogeneous and isotropic expanding universe.

- (a) Compute the Christoffel symbols for this metric.
- (b) Write down the equation of motion of a point particle in this metric in the absence of external forces and derive the time dependence of the magnitude of its physical three-velocity. Comment on your result.