PHYS 480/581
General Relativity

Homework Assignment 9

Due date: Monday 10/26/2026 8pm, submitted electronically on UNM Canvas

Question 1 (2 points).

Starting from the Riemann tensor in the weak field limit,

$$R_{\alpha\beta\mu\nu} = \frac{1}{2}\left(\partial_\beta\partial_\mu h_{\alpha\nu} + \partial_\alpha\partial_\nu h_{\beta\mu} - \partial_\alpha\partial_\mu h_{\beta\nu} - \partial_\beta\partial_\nu h_{\alpha\mu}\right), \tag{1}$$

show that the Ricci tensor takes the form

$$R_{\beta\nu} = \frac{1}{2}\left(-\eta^{\alpha\mu}\partial_\alpha\partial_\mu h_{\beta\nu} + \partial_\beta H_\nu + \partial_\nu H_\beta\right), \tag{2}$$

where

$$H_\nu = \eta^{\mu\alpha}\left(\partial_\mu h_{\alpha\nu} - \frac{1}{2}\partial_\nu h_{\alpha\mu}\right). \tag{3}$$

Question 2 (6 points).

Consider the Friedmann-Robertson-Lemaitre-Walker metric given by

$$ds^2 = -dt^2 + a^2(t)[dx^2 + dy^2 + dz^2] \tag{4}$$

where $a(t)$ is a function of coordinate time to be determined.

(a) Assuming that the stress-energy tensor is dominated by vacuum energy,

$$T_{\mu\nu} = -\frac{\Lambda}{8\pi G}g_{\mu\nu}, \tag{5}$$

use the Einstein equation to determine $a(t)$.

(b) Now, assume instead that the stress-energy tensor is dominated by nonrelativistic matter with zero pressure such that

$$T_{\mu\nu} = \begin{pmatrix} \rho_{\rm m}(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{6}$$

where $\rho_{\rm m}$ is the rest-frame energy density of the matter. Using the covariant conservation of the stress-energy tensor $\nabla_\mu T^{\mu\nu} = 0$, show that

$$\rho_{\rm m} \propto 1/a(t)^3. \tag{7}$$

(c) Using the solution given in Eq. (7), show that the Einstein equation implies that

$$a(t) \propto t^{2/3}, \tag{8}$$

for a universe dominated by nonrelativistic matter.

Question 3 (5 points).

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Then show that the circumference of a circle of radius R centered on the cosmic string and at $z = t =$ constant is smaller than $2\pi R$. The spacetime geometry around a cosmic string is thus said to have a *deficit angle*.