PHYS 480/581
General Relativity

Extra Problems #10

Question 1.

Let's take a very long thin solid rod of mass density λ (per unit length) extending along the z-axis (we can neglect the pressure of the rod). The rod is moving in the z direction with a small coordinate speed $V \ll 1$, making it essentially a static source. Using the non-relativistic and weak-field limit (i.e. $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $|h_{\mu\nu}| \ll 1$) of the geodesic equation

$$\frac{d^2x^i}{dt^2} \approx \frac{1}{2}\delta^{il}\partial_l h_{tt} + \delta^{il}\left(\partial_l h_{tj} - \partial_j h_{lt}\right) v^j, \tag{1}$$

compute the equation of motion for a non-relativistic particle propagating near this rod. In the weak, static limit, the Einstein equation admits the solution

$$h_{\beta\nu}(\vec{r}) = 2\int d^3r_{\rm s} \frac{G(2T_{\beta\nu} - \eta_{\beta\nu}T)}{|\vec{r} - \vec{r}_{\rm s}|}, \tag{2}$$

where the integral runs over where the stress-energy tensor has support (i.e. where the matter/energy is). Assume that the stress-energy tensor admits a perfect fluid form.

Question 2.

Show that $h_{\mu\nu} = A_{\mu\nu}\cos(\omega t - \vec{k}\cdot\vec{r})$ is a solution to the weak-field Einstein equation in vacuum $\Box^2 h_{\mu\nu} = 0$. Here, $A_{\mu\nu}$ is a constant matrix. How is ω related to k? What constraints does the requirement that $H_\nu = 0$ put on the matrix $A_{\mu\nu}$?