PHYS 480/581
General Relativity

Extra Problems #12

Question 1.

Imagine that in a certain region of spacetime, we have $H_{\mu\nu} = 0$ except for $H_{xx} = -H_{yy} = A\cos(\omega t - \omega z)$, where ω is a constant. This represents a plane wave whose crests move in the $+z$ direction at the speed of light.

(a) Show that this wave obeys the Lorenz gauge condition.

$$\partial_\mu H^{\mu\nu} = 0. \tag{1}$$

(b) Show that this wave is a solution to the weak-field Einstein equation in vacuum,

$$\Box^2 H^{\mu\nu} = 0. \tag{2}$$

(c) Determine the metric for the spacetime through which this gravitational wave moves.

(d) What condition must be satisfied at all points if we are to satisfy the weak-field condition $|h_{\mu\nu}| \ll 1$.