Solutions extra problems 12

Paul Steimle

a) show: $\partial_\mu H^{\mu\nu} = 0!$

$\partial_t H^{tt} = \partial_t 0 = 0$ $\qquad$ $\partial_y H^{yy} = \partial_y A\cos(\omega t - \omega z) = 0$

$\partial_x H^{xx} = \partial_x A\cos(\omega t - \omega z) = 0$ $\qquad$ $\partial_z H^{zz} = \partial_z 0 = 0$

$\Rightarrow \partial_\mu H^{\mu\nu} = 0!$

b) show: $\partial_\alpha \partial^\alpha H^{\mu\nu} = 0$

obv for $\mu = t;\ \nu = t$ and $\mu = z\ \nu = z$ $\quad \partial^\alpha \partial_\alpha H^{\mu\nu} = 0$ as $H^{tt} = H^{zz} = 0$

$\mu = x\ \nu = x$:

$$
\begin{aligned}
(\eta^{\alpha\beta} + h^{\alpha\beta})\,\partial_\alpha \partial_\beta H^{xx} &\approx \eta^{\alpha\beta}\,\partial_\alpha \partial_\beta H^{xx} \quad \text{for 1. order} \\
&= \eta^{tt}\,\partial_t \partial_t H^{xx} + \eta^{zz}\,\partial_z \partial_z H^{xx} \\
&= -1\,\partial_t^2 A\cos(\omega t - \omega z) + 1\,\partial_z^2 A\cos(\omega t - \omega z) \\
&= -A\left[-\cos(\omega t - \omega z)\right]\omega^2 + A\left[-\cos(\omega t - \omega z)\right]\omega^2 \\
&= 0
\end{aligned}
$$

same goes for $\mu = y;\ \nu = y$ $\Rightarrow$ $\Box^2 H^{\mu\nu} = 0!$

c) calculate the metric!

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ $\qquad h_{\mu\nu} = H_{\mu\nu} + \frac{1}{2}\eta^{\mu\nu} H$

$H = \eta^{\alpha\beta} H_{\alpha\beta} = \eta^{xx} H_{xx} + \eta^{yy} H_{yy} = 1 \cdot A\cos(\omega t - \omega z) + 1 \cdot [-A\cos(\omega t - \omega z)] = 0$

$\Rightarrow g_{\mu\nu} = \eta_{\mu\nu} + H_{\mu\nu} \Rightarrow ds^2 = -dt^2 + [1 + A\cos(\omega t - \omega z)]\,dx^2 + [1 - A\cos(\omega t - \omega z)]\,dy^2 + dz^2$

d) condition for $|h_{\mu\nu}| \ll 1$

as $\cos(\omega t - \omega z) = 1$ eventually we can only can choose A freely

$\Rightarrow$ for $|h_{\mu\nu}| \ll 1$; $|A| \ll 1$