PHYS 480/581
General Relativity

Extra Problems #13

Question 1.

Consider the following gravitational wave

$$h_{\mu\nu}^{\mathrm{TT}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & A\cos(k_\alpha x^\alpha) & A\sin(k_\alpha x^\alpha) & 0 \\ 0 & A\sin(k_\alpha x^\alpha) & -A\cos(k_\alpha x^\alpha) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{1}$$

where $k_\alpha = (-\omega, 0, 0, \omega)$ and A is a small number.

(a) What kind of polarization structure does this gravitational wave have? What would be the electromagnetic equivalent?

(b) Argue that such a wave perturbs a ring of particles in the xy plane in such a way that their shape becomes an ellipse that rotates in that plane. [*You may find the following trig identity useful:* $\cos A \cos B + \sin A \sin B = \cos(A - B)$.]

(c) What is the rotation rate of the ellipse in terms of ω? Does it rotate clockwise or counter-clockwise?