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Coffee Hour 13

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1 PROBLEM

1.1 Question:

What kind of polarization structure does this gravitational wave have? What would be the electromagnetic equivalent?

1.2 Answer:

This would be a circular wave. Its electromagnetic equivalent would be a circular polarized electromagnetic wave.

2 PROBLEM

2.1 Question:

Argue that such a wave perturbs a ring of particles in the xy-plane in such a way that their shape becomes an ellipse that rotates in that plane. You may find the following trig identity useful:

$$cosAcosB + sinAsinB = cos(AB) \tag{1}$$

2.2 Answer:

$$\begin{aligned}\Delta s^2 &= (\eta_{\mu\nu} + h^{TT}_{\mu\nu})\Delta x^\mu \Delta x^\nu \\ &= -\Delta t^2 + \Delta x^2 + (1 + Acos(k_\alpha x^\alpha))\Delta x^2 \\ &\quad + (1 - Acos(k_\alpha x^\alpha))\Delta y^2 + 2Asin(k_\alpha x^\alpha))\Delta x \Delta y\end{aligned} \tag{2}$$

With $\Delta t^2 = \Delta z^2 = 0$ and by switching to zylindrical coordinates, we obtain

$$\begin{aligned}\Delta s^2 = (1 + Acos(k_\alpha x^\alpha))R^2cos^2\theta + (1 - Acos(k_\alpha x^\alpha))R^2sin^2\theta \\ + 2Asin(k_\alpha x^\alpha)R^2 sin\theta cos\theta\end{aligned} \tag{3}$$

Furthermore, by employing the identities

$$sin^2A + cos^2A = 1 \tag{4}$$

$$cos^2A - sin^2A = cos(2A) \tag{5}$$

$$sinAcosA = \frac{sin(2A)}{2} \tag{6}$$

we end up with

$$\Delta s^2 = R^2\Big[1 + Acos(k_\alpha x^\alpha)cos(2\theta) + Asin(k_\alpha x^\alpha)sin(2\theta)\Big] \tag{7}$$

This now we can bring with Eq. 1 into its final form.

$$\boldsymbol{\Delta s^2} = R^2\Big[1 + A(cos(k_\alpha x^\alpha - 2\theta)\Big] = \boldsymbol{R^2\Big[1 + A(cos(\omega t - 2\theta)\Big]} \tag{8}$$

This equation now describes a ring that is perturbed in the xy-plane such that its shape becomes an ellipse that rotates within this plane.

3 PROBLEM

3.1 Question:

What is the rotation rate of the ellipse in terms of ω? Does it rotate clockwise or counter- clockwise?

3.2 Answer:

From Eq. 8 we can see:

$$\theta = \frac{\omega t}{2} \iff t = \frac{2t}{\omega} \tag{9}$$

Also from Eq. 8 we can obtain that it rotates counterclockwise.