PHYS 480/581
General Relativity

Extra Problems #14

Question 1.

Consider a universe with $\Omega_\Lambda > 1$ in which matter and radiation are negligible. Such a universe will never have a Big Bang singularity, but instead will have an instant of maximal (finite) density. Define that instant to be $t = 0$.

(a) Show that for such universe, $a(t) = b\cosh(\omega t)$, where $b = \sqrt{(\Omega_\Lambda - 1)/\Omega_\Lambda}$, and $\omega = H_0\sqrt{\Omega_\Lambda}$. If this universe is expanding at time t_0, will it ever cease expanding? If so, at what time?

(b) Imagine that observers in this universe determine from observations of their cosmic microwave background that $\Omega_\Lambda = 2$. How old is their universe at time t_0?

(c) Is the spatial geometry of this universe spherical, flat, or saddle-like?

(d) What is the curvature scale R of this universe (which is the scale over which the spatial curvature of the universe becomes evident)? Remember that we define $\Omega_K = -\kappa/H_0^2$ and $\kappa = \pm 1/R^2$.