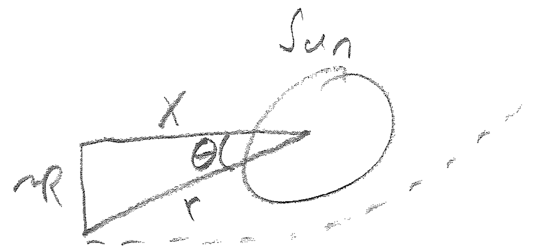
Extra Problem #1



$$\sin\theta = \frac{R}{r}$$

$$r = \sqrt{R^2 + X^2}$$

$$a = \frac{GM_\odot}{r^2}$$

$$a_y = \frac{GM_\odot}{r^2}\sin\theta = \frac{GM_\odot R}{r^3} = \frac{GM_\odot R}{(R^2+X^2)^{3/2}}$$

$$V_y = a_y t \Rightarrow dV_y = a_y dt$$

$$= \frac{GM_\odot R}{(R^2+X^2)^{3/2}}\,dt$$

Because we know that δ is small, we can assume that $\vec{V} \cong ct\hat{x}$ at all times, meaning that $X = ct$

$$\Rightarrow \quad dV_y = \frac{GM_\odot R}{(R^2+c^2t^2)^{3/2}}\,dt$$

$$\Rightarrow V_y = \int_{-\infty}^{\infty} \frac{GM_\odot R}{(R^2+c^2t^2)^{3/2}}\,dt$$

$$= GM_\odot R \int_{-\infty}^{\infty} \frac{1}{(R^2+c^2t^2)^{3/2}}\,dt$$

$$V_y = GM_{\odot}R\left(\frac{2}{R^2C}\right) = \frac{2GM_{\odot}}{RC}$$

$$\sin\delta = \frac{V_y}{C} = \frac{2GM_{\odot}}{RC^2}$$

$$\sin\delta \approx \delta = \frac{2GM_{\odot}}{RC^2} = 2.11E{-6} \text{ radians} \doteq 0.87 \text{ arcsec}$$