General Relativity
University of New Mexico
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GR Coffee hour 2

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1. *Question 1*

   Here we have two observers, each in an IRF. We can define one observer as stationary S and a second observer as moving at a constant velocity relative to the stationary observer S'. The formulation for proper time experienced by the moving observer during one lap around the torus is defined by

   $$\Delta\tau = \int \sqrt{-ds^2} = \int_{t_B}^{t_A} \sqrt{1 - v^2} dt$$

   where t_B is the time at which the moving frame first leaves the stationary frame, and t_A is the time when they reunite, from the stationary frame's perspective. Since we have a constant velocity of the moving frame around the torus for distance L, the integral becomes very straightforward

   $$\Delta\tau = \int_{t_B=0}^{t_A=\Delta t} \sqrt{1 - v^2} dt = \Delta t\sqrt{1 - v^2}$$

   where Δt is the total time elapsed from the time the moving frame left the stationary frame to the time they reunited. Additionally, due to the velocity of the moving frame being constant, we can define $\Delta t = L/v$ giving us

   $$\Delta\tau = \frac{L\sqrt{1 - v^2}}{v}$$

   Therefore, we see that the moving observer S' will experience a proper time that is shorter than the time experienced by the stationary observer for the time interval that it takes for the moving observer S' to travel all the way around the Torus and reconnect with observer S. This is consistent with Lorentz invariance, because the moving observer is traveling a shorter distance from their perspective, but at the same velocity, so they must experience a shorter amount of time passing.

2. *Question 2*

   Since observer $\bar{\mathcal{O}}$ observes the events in time order CBA (completely reversed from observer $\mathcal{O}$), then we see that the events AB and BC both have to have a spacetime separation that is "Spacelike". Therefore, $(\Delta S_{AB})^2 > 0$ and $(\Delta S_{BC})^2 > 0$, and subsequently

   $$\Delta t_{AB} < \Delta x_{BC}$$

   $$\Delta t_{BC} < \Delta x_{BC}$$

From the Lorentz transform, we see the time separation for 2 events with relative velocity β (relative to observer $\mathcal{O}$ who sees the evernts in time order ABC) is

$$\Delta t'_{AB} = \gamma(\Delta t_{AB} - \beta \Delta x_{AB})$$
$$\Delta t'_{BC} = \gamma(\Delta t_{BC} - \beta \Delta x_{BC})$$

In order to have some observer see the events occur in time order ACB, the frame's velocity must be large enough to flip the events B and C, but not high enough to flip events A and B. In other terms, $\Delta t'_{AB} > 0$ and $\Delta t'_{BC} < 0$. For this criteria to be met, from the lorentz transform we see that the following must also be met

$$\Delta t_{AB} > \beta \Delta x_{AB}$$

$$\Delta t_{BC} < \beta \Delta x_{BC}$$

It follows that

$$\frac{\Delta t_{AB}}{\Delta x_{AB}} > \beta$$
$$\frac{\Delta t_{BC}}{\Delta x_{BC}} < \beta$$

Therefore, as long as the velocity (β) of an observer relative to observer $\mathcal{O}$ is in the range

$$\frac{\Delta t_{AB}}{\Delta x_{AB}} > \beta > \frac{\Delta t_{BC}}{\Delta x_{BC}}$$

Then this observer will see the events occur in time order ACB.