

PHYS 480/581 General Relativity

Extra Problems #4

Question 1.

The electromagnetic Lagrangian is given by $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A_\mu J^\mu$, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and J^μ is the electric current.

(a) Show that

$$\frac{\partial F_{\alpha\beta}}{\partial(\partial_\mu A_\nu)} = \delta_\alpha^\mu \delta_\beta^\nu - \delta_\beta^\mu \delta_\alpha^\nu \quad (1)$$

(b) Use the above to show that

$$\frac{\partial(F_{\alpha\beta}F^{\alpha\beta})}{\partial(\partial_\mu A_\nu)} = 4F^{\mu\nu}. \quad (2)$$

(c) Use the fact that A_μ satisfies the Euler-Lagrange equation

$$\frac{\partial\mathcal{L}}{\partial A_\nu} - \partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu A_\nu)} \right) = 0, \quad (3)$$

and the results above to derive 2 of Maxwell's equations

$$\partial_\mu F^{\mu\nu} = -J^\nu. \quad (4)$$