The electromagnetic Lagrangian
d)

$$
\mathcal{L}=-\frac{1}{4} F_{\mu \nu} F^{\mu \nu}+A_{\mu} J^{\mu}, \quad F_{\mu \nu}=\partial_{\mu} A_{\nu}-\partial_{\nu} A_{\mu}
$$

we want to show $\frac{\partial F_{\alpha \beta}}{\partial\left(\partial_{\mu} A_{v}\right)}=\delta_{\alpha}^{\mu} \delta_{\beta}^{v}-\delta_{\beta}^{\mu} \delta_{\alpha}^{\nu}$

$$
\begin{aligned}
& \text { Plugging } F_{\alpha \beta}=\partial_{\alpha} A_{\beta}-\partial_{\beta} A_{\alpha} \text { into } \frac{\partial F_{\alpha \beta}}{\partial\left(\partial_{\mu} A_{\nu}\right)}, \\
& \frac{\partial F_{\alpha \beta}}{\partial\left(\partial_{\mu} A_{\nu}\right)}
\end{aligned}=\frac{\partial}{\partial\left(\partial_{\mu} A_{\nu}\right)}\left(\partial_{\alpha} A_{\beta}-\partial_{\beta} A_{\alpha}\right) \quad \begin{aligned}
& \text { Note that each term is on in } \\
& \text { nonzero if } \partial_{\alpha} A_{\beta}=\partial_{\mu} A_{\nu}, \text { or } \partial_{\beta} A_{\alpha}=d_{\mu} A_{\nu}, \\
& \text { in which case it is } 1 .
\end{aligned} \quad \begin{aligned}
& \text { in }
\end{aligned}
$$

6) We want to show $\frac{\partial\left(F_{\alpha \beta} F^{\alpha \beta}\right)}{\partial\left(\partial \mu A_{\nu}\right)}=\psi F^{\mu \nu}$

Applying the product rule gives
We can pull out two metrics to lower the indices in the

Now we realize that if we use the metrics to raise the indices on Fab, both terms are equal.

$$
\begin{aligned}
& \begin{aligned}
\frac{\partial\left(F_{\alpha \beta} F^{\alpha \beta}\right)}{\partial\left(\partial \mu A_{\nu}\right)} & =F_{\alpha \beta} \frac{\partial F^{\alpha \beta}}{\partial\left(\partial \mu A_{\gamma}\right)}+F^{\alpha \beta} \frac{\partial F_{\alpha \beta}}{\partial\left(\partial_{\mu} A_{\gamma}\right)}
\end{aligned}+\begin{array}{l}
\text { to } \\
\\
\end{array} \\
& \text { first term. } \\
& =2 F^{\alpha \beta} \frac{\partial F_{\alpha \beta}}{\partial\left(\partial_{\alpha} A v\right)} \quad \text { Plugging in } \frac{\partial F_{\alpha \beta}}{\partial(\text { dasiv })} \text { trot a) } \\
& =2 F^{\alpha \beta}\left(\delta_{\alpha}^{\mu} \delta_{\beta}^{\nu}-\delta_{\beta}^{\mu} \delta_{\alpha}^{2}\right) \\
& =2\left(F^{\mu \nu}-F^{\nu \mu}\right) \text { We know that } \quad F^{\mu \nu}=-F^{\nu \mu} \text {. } \\
& =4 F^{\mu \nu}
\end{aligned}
$$

c) we know $\frac{\partial \mathcal{L}}{\partial A_{\nu}}-\partial_{\mu}\left(\frac{\partial \mathcal{L}}{\partial\left(\partial_{\mu} A_{\nu}\right)}\right)=0$

We want to show $d_{\mu} F^{\mu \nu}=-J^{2}$

Let's Start by plugging $\mathcal{L}$ into the above equation.

$$
\begin{aligned}
& \frac{\partial}{\partial A_{\nu}}(\underbrace{-\frac{1}{4} F_{\alpha \beta} F^{\alpha \beta}}_{\substack{\text { This is independent } \\
\text { of } A_{\nu}}}+A_{\alpha} J^{\alpha})-\partial_{\mu}[\underbrace{\frac{\partial}{\partial\left(\partial_{\mu} A_{2}\right)}}_{\substack{\text { From part b), } \\
\text { when know this } \\
\text { is - Fut }}}(\underbrace{-\frac{1}{4} F_{\alpha \alpha} F^{\alpha \beta}}_{\substack{\text { This is in independent }}}+A_{\alpha} J^{\alpha})]=0 \\
& \begin{aligned}
& \Rightarrow \quad \frac{\partial}{\partial A_{\nu}}\left(A_{\alpha} J^{\alpha}\right) \\
& \delta_{\alpha}{ }^{\nu} J^{\alpha}+\partial_{\mu} F^{\mu \nu}
\end{aligned}=0 \quad \begin{array}{l}
\partial_{\mu} F^{\mu \nu}=-J^{\nu}
\end{array}
\end{aligned}
$$

