PHYS 480/581
General Relativity

Extra Problems #6

Question 1.

Let's consider the metric

$$ds^2 = -dt^2 + [f(q)]^2 dq^2, \tag{1}$$

where $f(q)$ is an arbitrary function of the spatial coordinate q.

(a) Derive both the t and q components of the geodesic equation, using the proper time τ as the independent variable.

(b) Show that the t component of the geodesic equation implies that

$$\frac{dt}{d\tau} = \text{constant}, \tag{2}$$

(c) From the q component of the geodesic equation, show that

$$f\frac{dq}{d\tau} = \text{constant}. \tag{3}$$

Hint: use the fact that $\mathbf{u} \cdot \mathbf{u} = -1$, *with* $\mathbf{u} \equiv dx^\mu/d\tau$. Use the above to argue that the trajectory of a free particle in this spacetime obeys

$$\frac{dq}{dt} = \frac{\text{constant}}{f}. \tag{4}$$

(d) Define a new coordinate system (t, x) with $x = F(q)$, where F is the antiderivative of $f(q)$ (that is, $dF/dq = f(q)$). Show that the metric given in Eq. (1) above, once transformed to the (t, x) coordinates, is simply the metric for flat (2D) spacetime.