

PHYS 480/581 General Relativity

Extra Problems #7

Question 1.

Consider the metric of a three-sphere with coordinates $x^\mu = (\psi, \theta, \phi)$

$$ds^2 = d\psi^2 + \sin^2 \psi (d\theta^2 + \sin^2 \theta d\phi^2) \quad (1)$$

(a) Calculate the Christoffel connection coefficients.

(b) Calculate the Riemann tensor components

$$R^\alpha{}_{\beta\mu\nu} = \partial_\mu \Gamma^\alpha_{\beta\nu} - \partial_\nu \Gamma^\alpha_{\beta\mu} + \Gamma^\alpha_{\mu\gamma} \Gamma^\gamma_{\beta\nu} - \Gamma^\alpha_{\nu\sigma} \Gamma^\sigma_{\beta\mu}. \quad (2)$$

(c) Compute the Ricci tensor $R_{\mu\nu}$ and scalar R

$$R_{\mu\nu} = R^\alpha{}_{\mu\alpha\nu}, \quad R = R^\mu{}_\mu. \quad (3)$$

(d) Show that

$$R_{\alpha\beta\mu\nu} = \frac{R}{6} (g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\nu} g_{\beta\mu}). \quad (4)$$

Spaces that satisfies this property are called *maximally symmetric*.