PHYS 480/581
General Relativity

Homework Assignment 1 Solutions

Question 1 (3 points).

Moore's Problem 1.2
Solutions:

(a) We are given the relativistic Doppler formula. In the limit that $v/c \ll 1$, we can Taylor expand it as

$$\frac{\lambda}{\lambda_0} = \frac{\sqrt{1 - v/c}}{\sqrt{1 + v/c}} \approx \frac{1 - (v/2c)}{1 + (v/2c)} \approx (1 - (v/2c))(1 - (v/2c)) = 1 - v/c + \mathcal{O}((v/c)^2), \quad (1)$$

where v here is the detector's speed relative to the laser at the time of emission. Now, write $\lambda = \lambda_0 - \Delta\lambda$ (since $\lambda < \lambda_0$, i.e. it's a blue shift). We then have

$$\frac{\lambda}{\lambda_0} = \frac{\lambda_0 - \Delta\lambda}{\lambda_0} \approx 1 - v/c \rightarrow \frac{\Delta\lambda}{\lambda_0} \approx \frac{v}{c}. \quad (2)$$

Now, if the distance between the laser and detector is d, then the time it takes for the photon to go from one to the other if $\Delta t = d/c$. The speed gained by the detector relative to the laser at emission is then $v = g\Delta t = gd/c$. We thus get

$$\frac{\Delta\lambda}{\lambda_0} \approx \frac{gd}{c^2} \quad (3)$$

(b) If $g = 9.8\,\mathrm{m/s^2}$ and $d = 25$ m, then we get

$$\frac{\Delta\lambda}{\lambda_0} = 2.7 \times 10^{-15}. \quad (4)$$

(c) The first thing we need is to compute the gravitational acceleration on the surface of the neutron star. This is given by

$$g_{\mathrm{NS}} = \frac{GM_{\mathrm{NS}}}{R_{\mathrm{NS}}^2}, \quad (5)$$

where G is Newton's gravitational constant. Putting $M_{\mathrm{NS}} = 3 \times 10^{30}$ kg and $R_{\mathrm{NS}} = 12$ km, we find $g_{\mathrm{NS}} = 1.39 \times 10^{12}\,\mathrm{m/s^2}$. In this case

$$\frac{\Delta\lambda}{\lambda_0} \approx \frac{g_{\mathrm{NS}}d}{c^2} = 3.9 \times 10^{-4}, \quad (6)$$

which is much larger than on Earth.

Question 2 (4 points).

Moore's Problem 1.3
Solutions:

(a) The time it takes for the photon to travel from one side of the laboratory to the other is

$$\Delta t = d/c. \tag{7}$$

According to the Equivalence Principle, in that time, the photon will "fall" (or rather, the lab would move up by) a distance

$$h = \frac{1}{2} g \Delta t^2 = \frac{g}{2}\left(\frac{d}{c}\right)^2, \tag{8}$$

using the standard classical mechanics expression. Putting some numbers here with $d = 3$ m, we get

$$h = \frac{g}{2}\left(\frac{d}{c}\right)^2 = 4.9 \times 10^{-16}\,\mathrm{m}, \tag{9}$$

which is about half the size of a proton, a very small deflection indeed. This could also be expressed in terms of a deflection angle, which in the small-angle approximation is given by

$$\theta \simeq \frac{h}{d} = 1.6 \times 10^{-16}\,\mathrm{rad}. \tag{10}$$

(b) From the previous problem, we know that $g_{\mathrm{NS}} = 1.39 \times 10^{12}\,\mathrm{m/s^2}$. We thus get

$$\frac{g_{\mathrm{NS}}}{2}\left(\frac{d}{c}\right)^2 = 6.96 \times 10^{-5}\,\mathrm{m}. \tag{11}$$

The deflection angle is then $\theta \simeq 2.3 \times 10^{-5}$ rad.

Question 3 (2 points).

Moore's Problem 2.7
Solutions: Assume that in the ground frame, the two lights (A and B) are separated by a distance $\Delta x_{\mathrm{AB}} = x_{\mathrm{B}} - x_{\mathrm{A}} > 0$, meaning that light B here is closer to the front of the train, which is moving in the positive x direction. In the ground frame, the two lights blink simultaneously so $\Delta t_{\mathrm{AB}} = t_{\mathrm{B}} - t_{\mathrm{A}} = 0$. In the rest frame of the train, these two intervals are

$$\begin{pmatrix} \Delta t'_{\mathrm{AB}} \\ \Delta x'_{\mathrm{AB}} \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} 0 \\ \Delta x_{\mathrm{AB}} \end{pmatrix}, \tag{12}$$

which means that $\Delta t'_{\mathrm{AB}} = t'_{\mathrm{B}} - t'_{\mathrm{A}} = -\beta\gamma\Delta x_{\mathrm{AB}} < 0$. This means that $t'_{\mathrm{B}} < t'_{\mathrm{A}}$, which implies that in the train frame the light closest to the front of the train blinks first.