PHYS 480/581
General Relativity

Homework Assignment 3

Due date: Wednesday 02/07/2024 5pm, submitted electronically on UNM Canvas

Question 1 (7 points).

Imagine we have a tensor (matrix) $X^{\mu\nu}$ and a vector V^μ, with components

$$X^{\mu\nu} = \begin{pmatrix} 2 & 0 & 1 & -1 \\ -1 & 0 & 3 & 2 \\ -1 & 1 & 0 & 0 \\ -2 & 1 & 1 & -2 \end{pmatrix}, \qquad V^\mu = (-1, 2, 0, -2)\,. \tag{1}$$

Assuming that these two objects live in flat spacetime with a Minkowski metric $\eta_{\mu\nu}$, find the components of:

(a) $X^\mu{}_\nu$

(b) $X_\mu{}^\nu$

(c) $X^{(\mu\nu)} \equiv \frac{1}{2}\left(X^{\mu\nu} + X^{\nu\mu}\right)$

(d) $X_{[\mu\nu]} \equiv \frac{1}{2}\left(X_{\mu\nu} - X_{\nu\mu}\right)$

(e) $X^\lambda{}_\lambda$

(f) $V^\mu V_\mu$

(g) $V_\mu X^{\mu\nu}$

Question 2 (2 points).

The electromagnetic Lagrangian density is $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$. With the help of Eq. (4.14) in Moore, write down $\mathcal{L}$ in terms of the $\vec{E}$ and $\vec{B}$ field components.

Question 3 (5 points).

Moore Problem 5.5