Physics 480/581
General Relativity

Homework Assignment 6

Due date: Wednesday 02/28/2024 5pm, submitted electronically on UNM Canvas

Question 1 (4 points).

Moore Problem 8.6

Question 2 (3 points).

Moore Problem 17.5

Note that the metric in the p, q coordinate system is

$$g'_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & (bq)^{-2} \end{pmatrix}. \tag{1}$$

Question 3 (2 points).

Show that if we impose the metric compatibility requirement

$$\nabla_\alpha g_{\mu\nu} = 0, \tag{2}$$

then the connection admits the standard Christoffel form

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu} \right). \tag{3}$$

Question 4 (2 points).

Show that the components of the covariant derivative of a vector A^ν

$$\nabla_\mu A^\nu = \partial_\mu A^\nu + \Gamma^\nu_{\mu\alpha} A^\alpha \tag{4}$$

transform like a tensor under the coordinate transformation $x^\mu \to x'^\mu$.