# PHYS 480/581
# General Relativity

## Homework Assignment 9

**Due date: Friday 3/29/2024 5pm, submitted electronically on UNM Canvas**

**Question 1** (6 points).

---

Consider the Friedmann-Robertson-Lemaitre-Walker metric given by

$$ds^2 = -dt^2 + a^2(t)[dx^2 + dy^2 + dz^2] \tag{1}$$

where $a(t)$ is a function of coordinate time to be determined.

**(a)** Assuming that the stress-energy tensor is dominated by vacuum energy,

$$T_{\mu\nu} = -\frac{\Lambda}{8\pi G} g_{\mu\nu}, \tag{2}$$

use the Einstein equation to determine $a(t)$.

**(b)** Now, assume instead that the stress-energy tensor is dominated by nonrelativistic matter with zero pressure such that

$$T_{\mu\nu} = \begin{pmatrix} \rho_{\rm m}(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{3}$$

where $\rho_{\rm m}$ is the rest-frame energy density of the matter. Using the covariant conservation of the stress-energy tensor $\nabla_\mu T^{\mu\nu} = 0$, show that

$$\rho_{\rm m} \propto 1/a(t)^3. \tag{4}$$

**(c)** Using the solution given in Eq. (4), show that the Einstein equation implies that

$$a(t) \propto t^{2/3}, \tag{5}$$

for a universe dominated by nonrelativistic matter.

**Question 2** (5 points).

---

Moore 23.6 a,c,d,e

Then show that the circumference of a circle of radius $R$ centered on the cosmic string and at $z = t =$ constant is smaller than $2\pi R$. The spacetime geometry around a cosmic string is thus said to have a *deficit angle*.