General Relativity

PHYS 480/581

Instructor Info

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Course Info

Prereq: A good foundation in classical mechanics, electromagnetism, vector calculus, and linear algebra; some knowledge of Special Relativity.

Mon & Wed

11:30am-12.45pm

PAIS 1140

Course Website

GR Coffee Hours

Mondays and Thursdays

3-4pm

PAIS 2210

TA Info

Kylar Greene

Friday 1:30-2:30pm

Zoom

kygreene@unm.edu

Why General Relativity?

General Relativity (GR) is one of the most beautiful theory ever discovered. At its core, it links a phenomenon that we all experience – gravity – to the nature of spacetime itself and the energy and matter it contain. Using an elegant mathematical framework, GR details both how the very fabric of spacetime reacts to the presence of large matter concentrations, and how the latter evolve within this curved spacetime. GR has been extremely successful at describing observations from a range of length scales spanning over 30 orders of magnitude from the size of the observable universe to tabletop-sized experiments here on Earth. Recently, the first direct detection of gravitational waves has allowed us to directly probe GR near the event horizon of a black hole. Join me on this epic journey to unveil the nature of spacetime itself!

Overview

This class is aimed at senior physics undergraduates and graduate students. In the first part of the course, we will establish the physical and mathematical language necessary to have grownup conversations about GR. This includes discussing four-vectors and index notations, reviewing Special Relativity and Lorentz transformation, introducing the metric and its related tensors, and discussing general coordinate systems. With this foundation established, the second part of the course will focus on the structure of curved spacetime and how to describe it with the tools and concepts introduced in the first part of the course. This will culminate with the derivation and interpretation of the Einstein equation, probably the most insightful equation ever derived. The third part of the course will cover applications of GR such as black holes, gravitational waves astronomy, and cosmology.

About Me

I am a practicing theoretical cosmologist/astrophysicist with a keen interest about dark matter and the physics of the early Universe. While I actually use GR very little in my day-to-day research nowadays, I must say that taking a GR course as an undergraduate had a huge impact on my decision to become a theoretical physicist. I hope that this course has a lasting impact on you too!

Material

Required Text

Moore, T.A. *A General Relativity Workbook*. 1st Edition. University Science Books. 2013. (ISBN: 978-1-891389-82-5, eBook ISBN: 978-1-938787-32-4)

Useful References

Carroll, S.M. *Spacetime and Geometry: An Introduction to General Relativity*. 1st Edition. Cambridge University Press. 2019. (ISBN: 978-1-108488-39-6)

Hartle, J. B. *Gravity: an Introduction to Einstein's General Relativity*, San Francisco: Addison-Wesley. 2003. (ISBN: 978-0-8053-8662-2)

Grading Scheme

5%	Presenting solutions to Moore's boxes
25%	Homework assignments
25%	Midterm exam
25%	Final exam
20%	Final project (15% term paper + 5% oral presentation)

FAQs

? I don't know any differential geometry. Is this a problem?

! Not at all. In my opinion, having strong physical intuition and good spatial visualization skills are much more important to understand GR. Remember, this is a physics class, not a math class.

? Is this a "hard" class?

! I will not sugarcoat this: this might be the most technically advanced class you have taken so far. However, we will go through the material slowly and together, and work out in groups as many problems as possible so that the concepts can really sink in.

? I am afraid of tensor calculus. What should I do?

! Tensor calculus has a bad reputation, but for no good reason. With a good knowledge of linear algebra and vector calculus, grasping the key concepts behind tensor calculus should be straightforward.

? Did Einstein really came up with all of this by himself?

! Yes, pretty much. The math used by GR (non-Euclidean geometry) had already been develop by Riemann years earlier. Einstein's genius was to connect this Riemannian geometry with the structure of spacetime he was trying to describe, resulting in the very elegant GR framework we have today.

Class Format

There will be assigned reading before each lecture (usually one short chapter in Moore). Students will be expected to have done the reading before class time and come prepared with questions, if any. In addition, "boxes" from each chapter of Moore's book will be assigned in advance to specific students. During class time, we will first briefly review the main concepts from the day's reading and answer any related questions. I will then ask the students with assigned "boxes" to present their solutions to the rest of the class using the blackboard. The rest of the class will be spent solving example problems either with the whole class, or in small group.

Final Project

Each student will choose a topic from a list of possible projects (to be provided at a later date) and write a ~ 2000-word (~ 2500 for graduate students) term paper on that topic. Your paper must show how GR is relevant for the phenomenon you are studying and be supported by the relevant calculations. During the last week of classes, you will be asked to do a short presentation ($\sim$10 mins) summarizing the content of your term paper.

Homework Assignment

There will be a series of homework assignments spread out over the course of the semester. They will be due every ~ 7 days. They will be posted on the course webpage. Homeworks are to be submitted online on UNM Canvas using the appropriate link provided there. Homework assignments submitted up to 24 hours after they are due will be accepted, but with a 30% penalty. Homework submitted after 24 hours will not be graded.

While I strongly encourage you to discuss the homework assignments with your classmates, the work you hand in must be entirely yours.

Undergraduate vs Graduate students

Graduate students (registered for PHYS 581) will be expected to demonstrate a more thorough understanding of the mathematical foundation of General Relativity. This will be assessed via some more advanced homework and exam problems, a more substantial term paper, as well as a more challenging final grading scheme.

GR discussion hours

We learn a lot by casually talking about physics with our peers. Much of the physics that I know I've learned that way. The GR discussion hour is my attempt to create a space where we (mostly you) can have casual conversations about GR and physics in general without the constraints of being in a formal class. Of course, you can show up to ask questions about the homework or the reading if you want. These will serve as my office hours. Otherwise, we will go through key GR-related problems or discuss more advanced topics that will (hopefully!) help you better understand what is "under the hood" of GR. If you have a spare credit hour, you can register for my PHYS 551 (graduate students) or PHYS 451 (undergraduate students) course using the CR/NC option. You will get the credit if you show up for at least 10 sessions.

Learning Goals

Upon successful completion of this course,...

- Students will understand the equivalence principle and its central role in General Relativity.
- Students will know and understand the properties of four-vectors and tensors and know how to manipulate them.
- Students will know what a geodesic is and how to compute a geodesic trajectory in a given spacetime.
- Students will understand the meaning of spacetime curvature, and how to assess whether a spacetime is curved.
- Students will understand the physical meaning of the Einstein field equations and discuss their properties.
- Students will understand what energy conservation means in a curved spacetime.
- Students will understand the properties of the Schwarzschild solution and how to use it to compute physical quantities.
- Students will know how to derive the equation of motion for gravitational waves (GW) and compute the expected GW signal far away from the source.
- Students will know how to compute distances and time intervals in an expanding Universe.
- Students will know how to relate the matter/energy content of the Universe to the Universe's expansion rate.
- Students will demonstrate their understanding of general relativity by applying learned concepts to a novel context.

Academic Integrity

Each student is expected to maintain the highest standards of honesty and integrity in academic and professional matters. The University reserves the right to take disciplinary action, up to and including dismissal, against any student who is found guilty of academic dishonesty or otherwise fails to meet the standards. Any student judged to have engaged in academic dishonesty in course work may receive a reduced or failing grade for the work in question and/or for the course. Academic dishonesty includes, but is not limited to, dishonesty in quizzes, tests, or assignments; claiming credit for work not done, done by others, or done by an Artificial Intelligence algorithms; hindering the academic work of other students; misrepresenting academic or professional qualifications within or without the University; and nondisclosure or misrepresentation in filling out applications or other University records.

Credit-hour statement

This is a three credit-hour course. Class meets for two 75-minute sessions of direct instruction for fifteen weeks during the Spring 2024 semester. Please plan for a minimum of six hours of out-of-class work (or homework, study, assignment completion, and class preparation) each week.

Inclusion and Respect

I consider this classroom to be a place where you will be treated with respect, and I welcome individuals of all ages, backgrounds, beliefs, ethnicities, genders, gender identities, gender expressions, national origins, religious affiliations, sexual orientations, ability - and other visible and non-visible differences. All members of this class are expected to contribute to a respectful, welcoming and inclusive environment for every other member of the class.

Citizenship and/or Immigration Status: All students are welcome in this class regardless of citizenship, residency, or immigration status. Your professor will respect your privacy if you choose to disclose your status. UNM as an institution has made a core commitment to the success of all our students, including members of our undocumented community. The Administration's welcome is found on our website.

Our classroom and our university should always be spaces of mutual respect, kindness, and support, without fear of discrimination, harassment, or violence. Should you ever need assistance or have concerns about incidents that violate this principle, please access the resources available to you on campus, especially the LoboRESPECT Advocacy Center and the support services listed on its website. Please note that, because UNM faculty, TAs, and GAs are considered "responsible employees" by the Department of Education, any disclosure of gender discrimination (including sexual harassment, sexual misconduct, and sexual violence) made to a faculty member, TA, or GA must be reported by that faculty member, TA, or GA to the university's Title IX coordinator at the Office of Compliance, Ethics, and Equal Opportunity. For more information on the campus policy regarding sexual misconduct, please see this link.

Accommodations for Students with Disabilities

UNM is committed to providing equitable access to learning opportunities for students with documented disabilities. As your instructor, it is my objective to facilitate an inclusive classroom setting, in which students have full access and opportunity to participate. To engage in a confidential conversation about the process for requesting reasonable accommodations for this class and/or program, please contact Accessibility Resource Center at arcsrvs@unm.edu or by phone at 505-277-3506. Support: Contact me at fycr@unm.edu or at my office hours, and also contact the Accessibility Resource Center.

Tentative Class Schedule

MODULE 1: The physical and mathematical language of GR

Week 1	Equivalence Principle and Review of Special Relativity	Moore Chs. 1-2
Week 2	Lorentz transformations and four-vectors	Moore Chs. 2-3
Week 3	Index Notation, Metric, and Arbitrary Coordinates	Moore Chs. 4-5
Week 4	Tensors	Moore Ch. 6
Week 5	Tensors and Classical Field Theory	Moore Chs. 6-7

MODULE 2: Curved Spacetime and the Einstein equation

Week 6	Geodesics and the Covariant Derivative	Moore Chs. 8, 17
Week 7	Curvature and Geodesic deviation	Moore Ch. 18
Week 8	Riemann and Ricci Tensors Midterm Exam 03/06	Moore Ch. 19
Week 9	Stress-Energy Tensor and the Einstein Equation	Moore Chs. 20-21
Week 10	The Einstein Equation and Schwarzschild solution	Moore Chs. 22-23

MODULE 3: Applications

Week 11	The Schwarzschild Solution and Black holes	Moore Chs. 9-10
Week 12	Black hole event horizon and Hawking radiation	Moore Chs. 14, 16
Week 13	Gauge freedom and gravitational waves	Moore Chs. 30-31
Week 14	Gravitational wave generation and energy	Moore Chs. 32-33

Week 15	Cosmology and the Hubble Expansion	Moore Chs. 25-27
Week 16	Final Thoughts and Oral Presentations	
	Final Projects due	Last day of class
Week 17	Final exam	