PHYS 480/581 General Relativity
Final Project Topics

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Please write an essay of about 2000 words (2500 for graduate students) on one of the topics below. The level should be such that you would feel comfortable presenting the paper to your classmate (which you will do!). Try to focus on the main points. How does the topic fit into the field of general relativity? What are the main issues, the main results and the important techniques used? **The essays are due on 05/03/2024 5:00pm, on Canvas.**

I give below a range of topics ranging from more mathematical ones to more applied or even pop culture projects. I provide below some references for each topics as a starting point, but you are strongly encouraged to search for and use other references to go deeper into the topic. You are also encouraged to pursue other topics beyond this list. Consult with me in advance. Once you've chosen a topic, please let me know by email. I won't allow more than 2 students to choose the same topic. First come, first serve.

1. **Hamiltonian Formulation of Gravity**
   References:
   - R. Wald, "General Relativity", Chapter 10.
   - C. Misner, K. Thorne and J. Wheeler, "Gravitation", Chapter 21.

   The Hamiltonian formulation of gravity is the basis for any attempt to canonically quantize gravity. This is the topic for those of you wanting to learn something about one approach to quantum gravity.

2. **First Detection of Gravitational Waves by LIGO (GW150914)**
   References:
   - B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), "Observation of Gravitational Waves from a Binary Black Hole Merger", Phys. Rev. Lett. 116, 061102, (2016).
   - B. P. Abbott et al. (LIGO Scientific and Virgo Collaborations), "Tests of General Relativity with GW150914", Phys. Rev. Lett. 116, 221101 (2016).
   - Many other papers on the LIGO website, under the header GW150914 at the bottom of the page.

   The detection of the binary black hole merger GW150914 marked the beginning of gravitational wave astronomy, and was the culmination of decades of experimental development aimed at detecting gravitational waves. Review the detailed the event, how the LIGO detected it, and the tests of General Relativity that this event allowed us to perform.

3. **Non-coordinate bases and spinors**
   References:
   - S. Carroll, "An Introduction to General Relativity: Spacetime and Geometry", Appendix F.
   - C. Misner, K. Thorne and J. Wheeler, "Gravitation", Chapter 41.

   In this course, we only consider coordinate bases to describe to describe vectors and tensors in spacetime. However, nothing stops us from using other bases not based on coordinates. This can in fact lead to a dramatic simplification of some GR calculations. Non-coordinate bases are also required to describe how spinors (describing spin-1/2 particles for instance) behave in curved spacetime. Describe the theory behind using a non-coordinate basis, discuss how GR is formulated in such a language, and describe how it can be used to described spinors in spacetime.

4. **Rotating Black Holes**
   References:
   - S. Carroll, "An Introduction to General Relativity: Spacetime and Geometry", Chapter 6, Section 6.5-6.6.
   - S. Hawking and G. Ellis, "The Large Scale Structure of Space-Time", Sections 5.5 & 5.6.
   - T. A. Moore, "A General Relativity Workbook", Chapters 36-39.

In class I will only have time to discuss the most simple black hole solutions, namely the Schwarzschild and Reissner-Nordstrom black holes. Rotating black holes are of interest in astrophysics and cosmology and have some rather different properties.

5. **The Science of Interstellar**
References:

- K. Thorne, "The Science of Interstellar", United States: W.W. Norton (2014).

The movie "Interstellar" by Christopher Nolan incorporates several elements for which General Relativity is required to physically describe what is happening on the screen. From the super massive spinning black hole Gargantua to the traversable wormhole and the extreme time dilation and tidal waves of Miller's planet, there is a lot to choose from. Pick one or two examples of General Relativistic effects from the movie and explain quantitatively (i.e. with math) how they work. Which effects are accurately represented in the movie?

6. **The First Discovery of a Binary Neutron Star Merger (GW170817)**
References:

- B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), "GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral", Phys. Rev. Lett. 119, 161101 (2017).
- B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), "Tests of General Relativity with GW170817", Phys. Rev. Lett. 123, 011102 (2019).
- Many other papers on the LIGO website, under the header GW170817near the center of the page.

The discovery of the binary neutron star merger GW170817 marked the true beginning of multimessenger astronomy. The observation of both gravitational waves and electromagnetic radiation from the same source allowed several tests of General Relativity to be performed. Describe the main features of this event, and quantitatively describe which aspects of General Relativity were tested in this event.

7. **Horndeski Gravity**
References:

- arXiv:1901.07183
- arXiv:1712.02710

Horndeski gravity (originally from Walter Horndeski, 1974) is the most general theory one can write down involving one additional scalar degree of freedom added to a metric theory of gravity in 4 dimensions, where the equations of motion are up to second order in derivatives. Describe the general form that the Lagrangian takes in this theory and summarize the current constraints on this modification of General Relativity.

8. **General Relativity and the NAVSTAR Global Positioning System (GPS)**
References:

- N. Ashby, "Relativity and the Global Positioning System", Physics Today, May 2002, 41.
- H. F. Fliegel and R. S. DiEsposti, "GPS AND RELATIVITY: AN ENGINEERING OVERVIEW" (1996). Available here.

We have come to rely on GPS for navigation in all sorts of situations. Determining your position on Earth to a precision of a few meters requires exquisite clock synchronization and a detailed knowledge of the spacetime metric near our planet. Maintaining the accuracy of the GPS over long time scale require General Relativity. Review the key calculations that enable the GPS to work.

9. **Double Pulsar PSR J1744-3922 as a Test of General Relativity**
References:

- R. P. Breton, V. M. Kaspi, M. Kramer, 2, M. A. McLaughlin, M. Lyutikov, S. M. Ransom, I. H. Stairs and R. D. Ferdman et al., "Relativistic Spin Precession in the Double Pulsar," Science 321, 104 (2008) [arXiv:0807.2644 [astro-ph]].
- R. P. Breton, V. M. Kaspi, M. Kramer, 2, M. A. McLaughlin, M. Lyutikov, S. M. Ransom, I. H. Stairs and R. D. Ferdman et al., "Using the double pulsar eclipses to probe fundamental physics," AIP Conf. Proc. 983, 469 (2008).

The double pulsar PSR J1744-3922 provides a new test of General Relativity. Explain the nature of the test.

10. **Einstein-Cartan Theory and Torsion**
References:

- A. Trautman, "Einstein-Cartan Theory", Encyclopedia of Mathematical Physics, Oxford: Elsevier, 2006, vol. 2, pages 189–195
- F. W. Hehl, P. von der Heyde, G. D. Kerlick, and J. M. Nester, "General relativity with spin and torsion: Foundations and prospects", Rev. Mod. Phys. 48, 393 (1976)

In the theory of General Relativity we will cover in class, we will assume that the Christoffel connection are symmetric in their two lower indices, resulting in a torsion-free theory. There is an alternate formulation of General Relativity that does not make this assumption and thus have a nonvanishing torsion tensor. This theory with nonzero torsion has come to be called the Einstein-Cartan theory. Describe the differences between Einstein-Cartan theory and General Relativity. In which context is it useful? Does it make different observational predictions?

11. **Early Tests of General Relativity**
References:

- S. Carroll, "An Introduction to General Relativity: Spacetime and Geometry", Chapter 5, Section 5.5.
- C. Misner, K. Thorne and J. Wheeler, "Gravitation", Chapters 38 and 40.

Early tests of General Relativity have helped the theory to become widely accepted within the scientific community. The precession of Mercury's perihelion and the deflection of light in the solar eclipse of 1919 were two early tests of GR that were widely publicized. Review the General Relativistic calculations that are necessary to correctly predict these effects, and compare them to the measurements made at the time. Please adopt an historical perspective.

12. **Closed Timelike Curves and the Movie "Primer"**
References:

- Primer (2004), Studio Canal.
- arXiv:1008.1127

Closed timelike curves (CTCs) are actual solutions to the Einstein field equations which would technically allow time travel. This is exploited in the cult movie "Primer" (2004), possibly the most realistic time travel movie ever made (if there is such a thing!), where one of the main character enters a CTC to communicate to his past self and make money on the stock market. With the help of spacetime diagram and some examples of spacetime metrics allowing CTCs, describe the features and problems (paradoxes) associated with these solutions. What kind of solutions have been proposed? How are these paradoxes handled in the movie?

13. **Modified Gravity and Dark Energy**
References:

- S. M. Carroll, V. Duvvuri, M. Trodden and M. S. Turner, Phys. Rev. D 70, 043528 (2004) [arXiv:astro-ph/0306438].
- A. Silvestri and M. Trodden, Rept. Prog. Phys. 72, 096901 (2009) [arXiv:0904.0024 [astro- ph.CO]].

Dark energy has emerged as one of the most pressing puzzles of fundamental physics. Is it due to a remnant cosmological constant, strange matter with an equation of state $p = -\rho$, or is it due to modified gravity? The topic of this essay will be to explore the third possibility. Specifically, consider the approach of replacing the gravitational Lagrangian R by $f(R)$, where f is some function.

14. **Wormholes**
References:

- Bambi & Stojkovic, *Astrophysical Wormholes*, arXiv:2105.00881.
- Maldacena & Milekhin, *Humanly traversable wormholes*, Phys. Rev. D **103**, 066007 (2021), arXiv:2008.06618.

Wormholes are hypothetical structure connecting different points in spacetime, are some of the strangest solutions to Einstein field equations. Briefly describe the history of wormholes, give their precise definition, and discuss quantitatively different example of wormholes. In which case are they traversable? Discuss their potential connection to time travel.

15. **Origin of Anisotropies in the Cosmic Microwave Background**
References:

- D. Scott, J. Silk and M. White, Science 268, 829 (1995).
- M. White, D. Scott and J. Silk, Ann. Rev. Astron. Astrophys. 32, 319 (1994).
- W. Hu, Univ. of Chicago website.
- D. Scott, Univ. of British Columbia website

With the discovery of cosmic microwave anisotropies (CMB) in 1992, the high precision data from the WMAP satellite and the SPT and ACT experiments, and culminating with the March 21 2013 first data release from the Planck satellite, the CMB has become the best explored window to probe the early universe and to constrain the parameters which describe our universe on large scales. Give a description of how density perturbations lead to fluctuations in the temperature of the CMB and why one can learn about various parameters which describe the composition of the universe using the angular power spectrum of the CMB.

16. **Gravity Probe B and Gravitomagnetism**
References:

- T. A. Moore, "A General Relativity Workbook", Chapter 35.
- Everitt et al. Phys. Rev. Lett. 106 (2011) 221101. arXiv:1105.3456.

In GR, the physics of gravity can display a behavior similar to that of magnetic field, leading to important physical such as the Lense-Thirring precession and geodetic precession. Present the mathematical framework and physical situations where these effects can occur, and how the spacecraft Gravity Probe B managed to measure these subtle effects.